A Note on sg* Closed Sets and their Separation Axioms

V. Ramya
Madurai Kamaraj University,

Abstract*: The aim of this paper is to introduce** sgclosed set in a Soft topological space and to study some of its properties. The concept** sg* **closure and interior operators are introduced. Then sg*soft open and closed sets and soft regular , soft normal sets are derived. Also their properties are derived. .**

Key-Words: sg* **open set,** sg* closed sets, sg*soft regular and sg* soft normal spaces

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1. INTRODUCTION

Molodtsov (1999) introduced the concept of soft set theory. Maji et.al. (2003) initiated the theoretical study on soft sets and investigated some of their properties. After that many suthors Maji & Ray (2002), Pei & Miao(2005), Cheng-Fu Yang (2008), Kong et. Al. (2008), Zou & Xiao (2008), Ge& Yang (2011) and Pei Wang & Jiali He (2015) studied the notion of soft sets and applied in many fields of science and engineering. Shabir & Naz (2011) and Cangmen et.al. (2011) initiated the concept of soft topology and investigated some of their properties. Later on, Min (2011), Hussian & Ahmad (2011) and Aygunoglu & Aygun (2012), Zorlutuna et.al (2012),Pazar Varol et.al. (2012), Nazmul & Samanta (2013), PaZAR Varol & Aygun (2013) and Geortiou & Megarities (2014) studied some of the properties of soft topological spcaces. Kannan (2012), Arockiarani & Arokia Lancy(2013), Chen (2013), Mahatma & Das (2014), Kandil et.al. (2014a), Guzel Ergul et.al. (2014), Kannan et.al. (2015), Akdag & Ozkan (2014d) and Yuksel et.al.(2014) generated some work forms of open and closed sets in soft topological spaces called soft g open sets, semi open soft sets, soft regular open sets, pre open soft sets (α −open sosft set, semi open soft sets, β − open soft sets) soft generalized pre regular closed sets, soft stringly g closed sets, soft b-open sets and soft regular generalized closed sets respectively in soft topological spaces and studied some of their basic properties.

Shabir & Naz (2011) introduced and investigated the corresponding separation axioms in soft soft toplogical speacs using the elements of x. Using the notion of soft points and soft elements, Cagman et.al. (2011), Hussian & Ahmad(2015) and Nazmul & Samanta (2014) initiated and investigated the corresponding separation axioms in soft topological spaces respectively. Kandil et.al. (2014b) introduced the concept of soft semi separation axioms using the elements of X. Akdag & Ozjan (2014c) introduced and studied the concept of soft pre separation axioms in soft topological spaces using the elements of X.

In this paper, the concept of sg* closed set is introduced in a soft topological space $(X, \tilde{\tau}\, E)$ and some of its properties are studied in section 2. Further, the concept of sg* closure and sg* interior operators are introduced and some of the fundamental properties are studied. The notion of $sg^{*}T_1$ $(i = 0, {}^{1}/_{2}, 1,2)$spaces are introduced in section 3 and characterized using sg* open and sg* soft closed sets in a soft topological space $(X, \tilde{\tau}\, E)$. Finally, sg* soft regular and sg* soft normal spaces are introduced and studied some of their basic soft topological properties in section 4.

2. s g* CLOSED SETS

2.2.1 Definition A soft set *(A,E)* is called sg* closed in a soft topological space $(X, \tilde{\tau}\, E)$ of $cl\,(A, E) \tilde{\subseteq} (U, E)$whenever $(A, E) \tilde{\subseteq} (U, E)$ and $(U, E)$is soft g open in $\tilde{X}$.

2.2.1 Let $X = \{a_1, a_2, a_3\}, E = \{b_1, b_2\}$and

$\tilde{\tau} = \{\tilde{\emptyset}, \tilde{X}, (A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E), (A_6, E), (A_7, E)\}$ where

$(A_1, E) = \{(b_1, \{a_2\}), (b_2, \{a_1\}),$ $(A_2, E) = \{(b_1, \{a_2\}), (b_2, X)\}$

$(A_3, E) = \{(b_1, \{a_{2,} a_3\}), (b_2, \{a_{2,} a_3\})\},$ $(A_4, E) = \{(b_1, \{a_{1,} a_3\}), (b_2, X)\},$

$(A_5, E) = \{(b_1, \emptyset)\{b_{2,}\{a_1\}\}$ $(A_6, E) = \{(b_1, \emptyset)\{b_{2,}\{a_{2,}a_3\}\}$ and

$(A_7, E) = \{(b_1, \emptyset), (b_2, X)\}$.

Clearly $(A, E) = = \{(b_1, \{a_{1,}a_3\})(b_2, \{a_3\})\}$ is sg* closed in $(X, \tilde{\tau}\ E)$.

since for *(A,E)* there exists a soft g open set $(U, E) = \{(b_1, \{a_{1,}a_3\}.(b_{2,}\{a_{2,}a_3\})$ such that $cl\ (A, E) \tilde{\subseteq} (U, E)$.

2.1 Theorem

Every soft closed set is sg* closed in a soft topological space $(X, \tilde{\tau}\ E)$.

Proof follows from 2.2.1. Definition and Theorem 1(3) (Shabir & Naz 2011)

2.2 Example

The following example shows that the converse of the above 2.2.1. Theorem need not be true.

Let $X = \{a_1, a_2, a_3\}, E = \{b_1, b_2\}$ & $\tilde{\tau} = \{\tilde{\emptyset}, \tilde{X}, (A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E)\}$

Where $(A_1, E) = \{(b_1, \{a_2\}), (b_2, \{a_1\}),$ $(A_2, E) = \{(b_1, \{a_3\}), (b_2, \{a_1, a_2\})\}$

$(A_3, E) = \{(b_1, \{a_{2,}a_3\}), (b_2, \{a_{1,}a_2\})\},$ $(A_4, E) = \{(b_1, X), (b_2, \{a_1, a_2\})\}$, and

$(A_5, E) = \{(b_1, \emptyset)\{b_{2,}\{a_1\})\}\}$ are soft open sets,

Here $(B, E) = \{(b_1, \{a_1\}), (b_2, \{a_{1,}a_2\})\}$ and $(C, E) = \{(b_1, \{a_1\}), (b_2, X)\ \}$ are sg* closed sets but are not soft closed sets.

2.3 Theorem

Every sg* closed set is soft g closed in the soft topological space $(X, \tilde{\tau}\ E)$.

Proof Let *(U,E)* be a sg* closed set and let *(B,E)* be a soft open set such that $(U, E) \tilde{\subseteq} (B, E)$.

By Theorem 4.2 (Kannan 2012). *(B,E)* is soft g open . Since *(U,E)* sg* closed, therefore $cl(U,E) \tilde{\subseteq} (B,E)$.

2.4 Theorem

Let $\{(U,E)_i : i \in J\}$be the collection of all sg* closed sets in a soft topological space $(X, \tilde{\tau}\, E)$. Then $\tilde{\cup}_{i\in j}\ (U,E)_i$ is also a sg* closed set in $(X, \tilde{\tau}\, E)$.

Proof. Let $\tilde{\cup}_{i\in j}\ (U,E)_i \tilde{\subseteq} (V,E)$ and (V,E) be the soft g open set. Since $(U,E)_i$ is sg* closed, then $cl(U,E)_i \tilde{\subseteq} (V,E)$ for each $i \in J$. Hence by Theorem 1(6) (Shabir& Naz 2011) $cl\tilde{\cup}_{i\in j}\ (U,E)_i \tilde{\subseteq} \tilde{\cup}_{i\in j}$ cl$(U,E)_i \tilde{\subseteq} (V,E)$.

The following example shows that the intersection of two sg* closed sets need not be sg* closed in a soft topological space $(X, \tilde{\tau}\, E)$.

2.5 Example

If (U,E) and (V,E) are two sg* closed sets in $(X, \tilde{\tau}\, E)$, then $(U,E) \tilde{\cap}\ (V,E)$ need not be a sg* closed in $(X, \tilde{\tau}\, E)$.

Let $X = \{a_1, a_2, a_3\}, E = \{b_1, b_2\}$ and

$\tilde{\tau} = \{\tilde{\emptyset}, \tilde{X}, (A_1,E), (A_2,E), (A_3,E), (A_4,E), (A_5,E), (A_6,E), (A_7,E)\}$ where

$(A_1,E) = \{(b_1,\{a_2\}), (b_2,\{a_1\}),$ $(A_2,E) = \{(b_1,\{a_2\}), (b_2,\{a_1,a_2\})\}$

$(A_3,E) = \{(b_1,\{a_{2,}a_3\}), (b_2,\{a_{1,}a_2\})\},$ $(A_4,E) = \{(b_1,\{a_{1,}a_3\}), (b_2,X)\},$

$(A_5,E) = \{(b_1,\{a_3\}), (b_2,\{a_1,a_3\})\}\, (A_6,E) = \{(b_1,\emptyset)\{b_{2,}\{a_1\}\}\}$

And $(A_7,E) = \{(b_1,\emptyset)\{b_{2,}\{a_1,\{a_2\}\}\}$

Clearly,$(U,E) = \{(b_1,\{a_1,a_3\}), (b_2,\{a_1,a_3\})\}$and $(V,E) = \{b_1,X), (b_2,\{a_1,a_3\})\}$are sg* closed sets in $(X, \tilde{\tau}\, E)$ but $(U,E) \tilde{\cap}\ (V,E) = \{(b_1,\{a_{1,}a_3\}), (b_{2,}\{a_2\})\}$ is not a sg* closed. Set.

2.6 Theorem

If (U,E) is sg* closed in a soft topological space $(X, \tilde{\tau}\, E)$ and $\tilde{\subseteq} (V,E) \tilde{\subseteq}\ cl(U,E)$ then (V,E) is sg* closed.

Proof Let $(V,E) \tilde{\subseteq}\ (A,E)$ and (A,E) be a soft g open set, Since (U,E) is a sg* closed, hence $cl(U,E) \tilde{\subseteq} (V,E)$. Therefore $cl(V,E) \tilde{\subseteq}\ cl(U,E) \tilde{\subseteq}\ (A,E)$.

2.7 Theorem

If a soft subset(U,E) is sg* closed in $(X, \tilde{\tau}\, E)$, then $cl(U,E) - (U,E)$ does not contain any non empty soft g closed set.

Proof. Let ((F,E) ve a soft g closed set such that $(F,E) \tilde{\subseteq}\ cl(U,E) - (U,E)$. Then $(F,E) \tilde{\subseteq}\ \tilde{X} - (U,E)$ Implies that $(U,E) \tilde{\subseteq}\ \tilde{X} - (F,E))$. Since (U,E) is sg* closed , then $cl(U,E) \tilde{\subseteq}\ \tilde{X} - (F,E))$. That is $(F,E) \tilde{\subseteq}\ \tilde{X} - c\,l(U,E)))$. Hence $(F,E) \tilde{\subseteq}\ cl(U,E) \tilde{\cap} \left(\tilde{X} - \left(cl(U,E)\right)\right) = \tilde{\emptyset}$.

2.8 Theorem

In a soft topological space $(X, \tilde{\tau}\, E)$, either $(A_e^x)$is soft g closed or $\tilde{X} - (A_e^x)$ is sg* closed.

Proof

Suppose that (A_e^x) is not soft g closed. Then $\tilde{X} - (A_e^x)$ is not soft g open set. This implies that $\tilde{X}$ Is the only soft open set containing $\tilde{X} - (A_e^x)$. Hence $\tilde{X} - (A_e^x)$ is a sg* closed.

3. SEPARATION AXIMOS

3.1 Definition A soft topological space $(X, \tilde{\tau}\, E)$ is said to $sg^*\ T_\circ$ space if for pair of distinct points $x, y \in X$, there exists a sg* open set (A,E) such that either $x\ \tilde{\in}\ (A,E)\ or\ x\ \tilde{\in}\ (A,E)$ and $y\ \tilde{\in}\ (A,E)$.

3.2 Definition

A soft topological space $(X, \tilde{\tau}\, E)$ is said to be $sg^*\ T_1$ space if for each pair of distinct points $x, y\ \in X$, there exists sg* open sets (A,E) and (B,E) such that $x\ \tilde{\in}\ (A, E)$ but $y\ \tilde{\notin}\ (B, E)$ and $x\ \tilde{\in}\ (B, E)$ but $y\ \tilde{\notin}\ (B, E)$.

3.3 Theorem

Let $(X, \tilde{\tau}\, E)$ be a soft topological space, (U,E) be a soft set in X and $x \in X$. Then the following statements hold good.

i) $x\ \tilde{\in}\ (U, E)$ if and only if $(x\, E) \tilde{\subseteq} (U, E)$;

ii) If $(x\, E)\ \tilde{\cap}\ (U, E) = \tilde{\emptyset}$, then $x\ \tilde{\notin}\ (U, E)$.

Proof (i) and (ii) follows from Definition 13 (Shabir & Naz 2011).

3.4 Theorem

Let $x, y \in X$ be distinct points, If there exist sg* open sets *(A,E)* and (B,E) in $\tilde{X}$ such that $x\ \tilde{\in} (A, E)$ and $y\ \tilde{\in} \tilde{X} - (A, E)$ and $y\ \tilde{\in}\ (B, E)$ and $x\ \tilde{\in} \tilde{X} - (B, E)$, then the soft topological space $(X, \tilde{\tau}\, E)$ is $sg^*\ T_\circ$ and $(X, T_\circ)$ is a $T_\circ$ space for each $\alpha \in E$.

Proof

Let $x, y \in X$ be distinct points and *(A,E)* and *(B,E)* be sg* open sets in $\tilde{X}$ such that $x \in (A, E)$ and $y\ \tilde{\in} \tilde{X} - (A, E)$ or $y\ \tilde{\in} (B, E)$ and $x\ \tilde{\in} \tilde{X} - (B, E)$. Hence $(X, \tilde{\tau}\, E)$ is $sg^*\ T_\circ$ space. By proposition 5 (Shabir & Naz 2011) for each $i \in E, (X, \tau_i\ )$ is a topological space , hence $x \in A(i)$and $y \notin A(i)$ or $x \in B(i)$ and $x \notin B(i)$. Therefore $(X, \tau_i\ )$ is a $T_\circ$ space for each $i \in E$.

3.5 Theorem

Let $x, y \in X$ be distinct points. If there exists sg* open set *(A,E)* and *(B,E)* in $\tilde{X}$ such that $x\ \tilde{\in} (A, E)\ \ y\ \tilde{\in} \tilde{X}(A, E)$ and $y\ \tilde{\in} \tilde{X} - (A, E)$ and $y\ \tilde{\in}\ (B, E)$ and $x\ \tilde{\in} \tilde{X} -$

(B,E), then the soft topological space $(X,\tilde{\tau}\, E)$ is $sg^*\ T_1$ and (X,τ_1) is T_1 space for each $i \in E$.

Proof Ananlogues to the proof of 2.3.2 Theorem.

3.6 Theorem

If (x,E) is sg* closed set for each $x \in X$,then the soft topological space $(X,\tilde{\tau}\, E)$ is $sg^*\ T_1$.

Proof

Let for each $x \in X$, (x,E)be a sg* closed set and $xy \in X$ such that $x \neq y$. Then $\tilde{X} - (x,E)$ is a sg* open set such that $y \tilde{\in} \tilde{X} - (x,E)$ and $x \tilde{\notin} \tilde{X} - (x,E)$. Similarly $\tilde{X} - (y,E)$ is sg* open set such that $x \tilde{\notin} \tilde{X} - (y,E)$ and $y \tilde{\notin} \tilde{X} - (y,E)$. Hence $(X,\tilde{\tau}\, E)$ is a $sg^*\ T_1$space.

Remark

If (x,E) is a soft closed set for each $x \in X$ then the soft topological space $(X,\tilde{\tau}\, E)$ is a $sg^*\ T_1$ space.

Proof follows From 2.2.1 Theorem and 2.3.4 Theorem.

3.7 Theorem

Let $(X,\tilde{\tau}\, E)$ be a soft topological space and Z be a non empty subset of X. If $(X,\tilde{\tau}\, E)$ is $sg^*\ T_0$ space then $(Z,\widetilde{\tau_{\tilde{Z}}}\, E)$ is $sg^*\ T_0$ space.

Proof

Let $x, y \in X$ be distinct points. Then by assumption, there exist sg* open sets (A,E) and (B,E) in $\tilde{X}$ such that $x \tilde{\in} (A,E)$and $z \tilde{\notin} (A,E)$ or $z \tilde{\notin} (B,E)$ and $x \tilde{\notin} (B,E)$. since $z \in Z$, implies that $z \tilde{\in} \tilde{Z}$. Hence $z \tilde{\in} \tilde{Z} \tilde{\cap} (B,E) = ({}^{z}B,E)$. Since $x \tilde{\in} (A,E)$, then $x \tilde{\in} B(e)$ for all $e \in E$, or $x \tilde{\notin} B(e)$ for some $e \in E$. If $x \tilde{\in} B(e)$ for all $e \in E$, then $x \tilde{\notin} \tilde{Z} \tilde{\cap} (B,E) = ({}^{\tilde{Z}}B,E)$. If $x \tilde{\notin} B(e)$ for some $e \in E$, then

$x \ \widetilde{\notin} \ \tilde{Z} \ \widetilde{\cap} \, B(e)$ .Hence $x \ \widetilde{\notin} \ \tilde{Z} \ \widetilde{\cap} \ (B,E) = (^{\tilde{Z}} B,E)$. Similarly, for $x \ \widetilde{\in} \ (A,E) \ and$ $z \ \widetilde{\in} \ (A,E)$, $x \ \widetilde{\notin} \ \tilde{Z} \ \widetilde{\cap} \ (A,E) = \ (^{\tilde{Z}} A,E)$ and $z \ \widetilde{\notin} \ \tilde{Z} \ \cap (B,E) = (^{\tilde{Z}} B,E)$. Hence $(Z, \widetilde{\tau_{\tilde{Z}}} \, E)$ is a $sg^* \ T_0$ space.

4. sg* REGULAR AND sg* NORMAL SPACES

4.1 Definition

A soft topological space $(X, \tilde{\tau} \, E)$ is said to be sg* regular if for each sg* closed set *(A,E)* and each point $x \,\widetilde{\in}\, \tilde{X} - (A,E)$ there exist disjoint sg* open sets $(B_{1,}E)$ and $(B_{2,}E)$such that $x \,\widetilde{\in}\, ((B_{1,}E), (A,E) \,\widetilde{\subseteq}\, (B_{1,}E) \,\widetilde{\cap}\, (B_{2,}E) = \widetilde{\emptyset}$.

4.2 Theorem

For a soft topological space $(X, \tilde{\tau} \, E)$, the following statements are equivalent.

i) $(X, \tilde{\tau} \, E)$ is sg* regular.

ii) For each $x \ \in X$ and each $(A,E) \ \widetilde{\in} \ sg^*O(X.x)$, there exists a $(B,E) \ \widetilde{\in} \ sg^*O(X.x)$ such that $x \ \widetilde{\in} \ (B,E) \,\widetilde{\subseteq}\, sg^* \, cl \, (B,E) \,\widetilde{\subseteq}\, (A,E)$.

iii) For each sg* closed set (F,E) of $\tilde{X}$,

$$(F,E) = \widetilde{\cap} \ \{sg^* \, cl \, (B,E) : \widetilde{\subseteq} \ (B,E), (B,E) \,\widetilde{\in}\, sg^*O \left(\tilde{X}\right)\}$$

iv) For each subset (U,E) of $\tilde{X}$ and each $(A,E) \,\widetilde{\in}\, sg^*O \left(\tilde{X}\right)$ with $(U,E) \ \widetilde{\cap} \, (A,E) \neq \widetilde{\emptyset}$, there exists $(B,E) \,\widetilde{\in}\, sg^*O \left(\tilde{X}\right)$ such that $(U,E) \ \widetilde{\cap} \, (B,E) \neq \widetilde{\emptyset}$, and $sg^* \, cl \, (B,E) : \widetilde{\subseteq} \ (A,E)$.

v) For each non-empty subset (U,E) of $\tilde{X}$ and each sg* closed set (F,E) of $\tilde{X}$ with $(U,E) \ \widetilde{\cap} \, (F,E) \neq \widetilde{\emptyset}$, there exists $(B,E), (W,E) \,\widetilde{\in}\, sg^*O \left(\tilde{X}\right)$ such that $(U,E) \ \widetilde{\cap} \, (B,E) \neq \widetilde{\emptyset}$, $(F,E) \,\widetilde{\subseteq}\, (W,E)$ and $(W,E) \,\widetilde{\cap}\, (B,E) \neq \widetilde{\emptyset}$.

Proof

(i) $\longrightarrow$ (ii)

Let $(A,E) \tilde{\in}\ sg^{*}O\left(\tilde{X},x\right)$.

Then $x\ \tilde{\notin}\ \tilde{X}-(A,E)$ and there exists $(C,E),(B,E) \tilde{\in}\ sg^{*}O\left(\tilde{X}\right)$ such that $\tilde{X}-(A,E) \tilde{\subseteq}\ (C,E), x\ \tilde{\notin}\ (B,E)$ and $(C,E)\ \tilde{\cap}\ (B,E) \neq \tilde{\emptyset}$. Therefore

$(B,E) \tilde{\subseteq}\ \tilde{X}-(C,E)$ and $x\ \tilde{\notin}\ (B,E) \tilde{\subseteq}\ sg^{*}\, cl\,(B,E)\ \tilde{\subseteq}\ \left(\tilde{X}-(C,E)\right) \tilde{\subseteq}\ (A,E)$.

(ii) $\longrightarrow$ (iii)

Let $\left(\tilde{X}-(F,E)\right) \tilde{\in}\ sg^{*}O\left(\tilde{X},x\right)$.

Then by (ii) there exists $(A,E) \tilde{\in}\ sg^{*}O\left(\tilde{X}\ x\right)$ such that

$x\ \ \tilde{\in}\ (A,E) \tilde{\subseteq}\ sg^{*}\, cl\,(A,E)\ \tilde{\subseteq}\ \left(\tilde{X}-(F,E)\right)$. Hence $(F,E) \tilde{\subseteq}\ \tilde{X}-sg^{*}\, cl(A,E)$. By taking $\tilde{X}-sg^{*}\, cl(A,E)=(B,E)$, then $(B,E) \tilde{\in}\ sg^{*}O(X)$ and $(U,E)\ \tilde{\cap}\,(B,E) \neq \tilde{\emptyset}$. Then $\tilde{X}-(A,E)$ is the sg* closed set containing *(B,E).*

Therefore $sg^{*}\, cl(B,E)\ \tilde{\subseteq}\ \tilde{X}-(A,E)$. Hence $x\ \ \tilde{\in}\, sg^{*}\, cl(B,E)$.

Therefore, $(F,E)=\ \tilde{\bigcap}\,\{sg^{*}\, cl(B,E){:}\,(F,E) \tilde{\subseteq}\ (B,E),(B,E) \tilde{\in}\ sg^{*}O\left(\tilde{X}\right)\}$

(iii) $\longrightarrow$ (iv)

Let $(A,E) \tilde{\in}\ sg^{*}O\left(\tilde{X}\ \right)$ with $x\ \ \tilde{\in}\ (A,E)\,\tilde{\cap}\,(U,E)$. Then $x\ \tilde{\notin}\ \tilde{X}-(A,E)$ and hence by (iii), there exists a sg* open set (W,E) such that $\tilde{X}-(A,E) \tilde{\subseteq}\ (W,E)$ and $x\ \ \tilde{\in}\ sg^{*}\, cl(W,E)$. By taking $(B,E)=\tilde{X}-\ sg^{*}\, cl(W,E)$, then (B,E) is a sg* open set and hence $(B,E)\ \tilde{\cap}\,(U,E) \neq \tilde{\emptyset}$. Hence $(B,E) \tilde{\subseteq}\ (\tilde{X}-(W,E))$ and $sg^{*}\, cl(B,E) \tilde{\subseteq}\ \tilde{X}-(W,E) \tilde{\subseteq}\ (A,E)$.

(iv) $\longrightarrow$ (v)

Let (F,E) be a sg* closed set. Then $(\tilde{X}-(F,E))$ is a sg* open and $(\tilde{X}-(F,E))\ \tilde{\cap}\,(U,E) \neq \tilde{\emptyset}$. Then there exists $(B,E) \tilde{\in}\ sg^{*}O\left(\tilde{X}\right)$ such that

$(U,E)) \,\tilde{\cap}\, (B,E) \neq \tilde{\emptyset}$ and $sg^{*}\, cl(B,E) \,\tilde{\subseteq}\, \tilde{X} - (F,E)$. Then by considering $(W,E) = \tilde{X} - \; sg^{*}\, cl(B,E)$ then $(F,E) \,\tilde{\subseteq}\, (W,E)$ and $(W,E)) \,\tilde{\cap}\, (B,E) \neq \tilde{\emptyset}$.

(v) $\longrightarrow$ (i)

sLet (F,E) be a sg* closed set and $x \;\tilde{\in}\; \tilde{X} - (F,E)$, then by (v) there exists $(W,E),(B,E) \;\tilde{\in}\; sg^{*}O\left(\tilde{X}\right)$ such that $(F,E) \,\tilde{\subseteq}\, (W,E)$ and $x \;\tilde{\in}\; (B,E)$ and $(W,E)) \,\tilde{\cap}\, (B,E) \neq \tilde{\emptyset}$

4.3 Theorem

Let $(X, \tilde{\tau}\, E)$ be a sot topological space and $x \in X$. If $(X, \tilde{\tau}\, E)$ is sg* regular space then, i) $x \in (U,E)$ if and only if $(x,E) \,\tilde{\cap}\, (U,E) \neq \tilde{\emptyset}$ for every sg* closed set (U,E).

ii) $x \;\tilde{\notin}\; (V,E)$ if and only if $(x,E) \,\tilde{\cap}\, (V,E) \neq \tilde{\emptyset}$ for every sg* open set (V,E).

Proof

(i) Given *(U,E)* is a sg* closed set such that $x \;\tilde{\notin}\; (U,E)$. Then there exist sg* open sets *(A,E)* and (B,E) such that $x \;\tilde{\in}\; (A,E)$ and $(U,E) \,\tilde{\subseteq}\, (B,E)$. Hence From 2.3.1 (i) Theorem $(x\, E) \,\tilde{\subseteq}\, (A,E)$. Hence $(U,E) \,\tilde{\cap}\, (x,E) \neq \tilde{\emptyset}$.

Conversely, if $(U,E) \,\tilde{\cap}\, (x,E) \neq \tilde{\emptyset}$, then from 2.3.1 (ii) Theorem, $x \;\tilde{\notin}\; (U,E)$.

(ii) Given *(V,E)* is a sg* open set such that $x \;\tilde{\notin}\; (U,E)$. If $x \;\tilde{\notin}\; U(e)$ for all $e \in E$, then $(x,E) \,\tilde{\cap}\, (U,E) = \tilde{\emptyset}$. If $x \;\tilde{\notin}\; U(e_1)$ and $x \;\tilde{\notin}\; U(e_2)$ for some $e_1, e_2, \in E$ then $x \in X - U(e_1)$ and $x \notin X - U(e_2)$ for some $e_1, e_2, \in E$. Hence $(U,E) \,\tilde{\cap}\, (x,E) \neq \tilde{\emptyset}$ and $\tilde{X} - (U,E)$ is sg* closed set such that $x \;\tilde{\notin}\; \tilde{X} - (U,E)$. Therefore from (i) $(x,E) \,\tilde{\cap}\, \left(\tilde{X} - (U,E)\right) = \emptyset$, hence $(x,E) \,\tilde{\subseteq}\, (U,E)$ and $x \;\tilde{\in}\; (U,E)$ which is a contradiction. Conversely, if $(U,E) \,\tilde{\cap} - (x,E) = \emptyset$, then from 2.3.1 (ii) Theorem $x \;\tilde{\in}\; (U,E)$.

4.4 Theorem

A soft topological space $(X, \tilde{\tau}\, E)$ is sg* regular if and only if for every sg* closed set *(A,E)* such that $(x, E) \tilde{\cap}\ (A, E) = \emptyset$, there exist disjoint sg* open sets $(A_{1,}, E)$ and $(A_{2,}, E)$ such that $(x, E) \tilde{\subseteq} \left(A_{1,}, E\right), (A, E) \tilde{\subseteq} \left(A_{2,}, E\right)$.

Proof

Suppose that $(X, \tilde{\tau}\, E)$ is sg* regular space and (A,E) is a sg* closed set such that $(x, E) \tilde{\cap}\ (A, E) = \emptyset$. Then from 2.3.1 (ii) Theorem $x\ \tilde{\notin}\ (A, E)$.

Hence there exist disjoint sg* open sets $(A_{1,}, E)$ and $(A_{2,}, E)$ such that $(x, E) \tilde{\subseteq} \left(A_{1,}, E\right), (A, E) \tilde{\subseteq} \left(A_{2,}, E\right)$.

Conversely, if *(A,E)* is a sg* closed set such that $(x, E) \tilde{\cap}\ (A, E) = \emptyset$, there disjoint sg* open set $(A_{1,}, E)$ and $(A_{2,}, E)$ and $\left(A_{1,}, E\right) \tilde{\cap}\ (A_{2,}, E) \neq \tilde{\emptyset}$.

4.5 Definition

A soft topological space $(X, \tilde{\tau}\, E)$ is said to be sg* normal if for any pair of disjoint sh* closed sets *(U,E)* and *(V,E*), there exist disjoint sg* open sets $\left(A_{1,}, E\right)$ and $(A_{2,}, E)$ such that $(U, E) \tilde{\subseteq} \left(A_{1,}, E\right)$ and $(V, E) \tilde{\subseteq} \left(A_{2,}, E\right)$.

4.6 Theorem

For a soft topological space $(X, \tilde{\tau}\, E)$ the following statements are equivalent.

i) $(X, \tilde{\tau}\, E)$ is sg* normal

ii) for any sg* closed set (*U,E*) and any sg* open set (*A,E*) containing (*U,E*), there exists a sg* open set (*B,E*) containing (*U,E*) such that

$$sg^*\, cl\, (B, E) \tilde{\subseteq}\ (A, E)$$

Proof (i) $\longrightarrow$ (ii) Let (*A,E*) be any sg* open set containing the sg* closed set (*U,E*). Then $\tilde{X} - (A, E)$ is a sg* closed set. By (i) there exist disjoint sg* open sets (*B,E*) and

(*C.E*) such that $(U,E) \tilde{\subseteq} (B,E)$ and $\tilde{X}-(A,E) \tilde{\subseteq} (C,E)$. Hence $\tilde{X}-(A,E) \tilde{\subseteq} sg^{*}int\ (C,E)$. Since $(B,E) \tilde{\cap} sg^{*}int\ (C,E) = \tilde{\emptyset}$, then $sg^{*}cl\ (B,E) \tilde{\cap} sg^{*}int\ (C,E) = \tilde{\emptyset}$ and $sg^{*}cl\ (B,E) = \tilde{X} - sg^{*}int\ (C,E) \tilde{\subseteq} (U,E)$. Therefore $(U,E) \tilde{\subseteq} (B,E) \tilde{\subseteq} sg^{*}cl\ (B,E) \tilde{\subseteq} (A,E)$.

(ii) $\longrightarrow$ (i) Let (*U,E*) and (*V,E*) be any disjoint sg* closed set of $\tilde{X}$, Since $\tilde{X}-(V,E)$ is a sg* open set containing (*U,E*), there exists a sg* open set (*B,E*) containing (*U,E*) such that $sg^{*}cl\ (B,E) \tilde{\subseteq} \tilde{X}-(V,E)$

s $(U,E) \tilde{\subseteq} sg^{*}int\ (B,E)$. By taking $(A,E) = sg^{*}int\ (B,E)$ and $(C,E) = \tilde{X} - sg^{*}cl\,(B,E)$, then (*A,E*) and (C,E) are disjoint sg* open sets such that $(U,E) \tilde{\subseteq} (A,E)$ and $(V,E) \tilde{\subseteq} (C,E)$. Therefore $(X,\tilde{\tau}\,E)$ is a sg* normal.

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