ANALYSIS OF UNIFORM INFINITE FIN BY ELZAKI TRANSFORM

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Abstract: The amount of heat convected from the fin surfaces has been determined by solving the general differential equation describing heat dissipation from the infinite fin via the calculus approach. Heat transfers by desirable quality of temperature gradient and the modes which transfer heat from one part of the medium to another are conduction, convection, and radiation. This paper is presenting the use of a Elzaki Transform for the analysis of uniform infinite fin by solving the general form of energy equation describing the heat dissipation from the surface of the medium and obtaining the distribution of temperature and hence the rate of heat convected into the surroundings from an infinite uniform fin.

Keywords: Heat convected, Uniform Infinite Fin, Elzaki Transform.

INTRODUCTION

Elzaki Transformation applied in solving boundary value problems in most of the science and engineering disciplines [1, 2, 3, 4, 5, 6, 7]. It also comes out to be very effective tool to analyze differential equations, Simultaneous differential equations, Integral equations etc. [7, 8, 9, 10, 11, 12, 13, 14]. spines are the extended surfaces projected from heat-conducting surfaces to improve the heat dissipation into the surroundings [1-3]. Fourier's law expressed as $\mathbf{H} = -KA\frac{d}{dy},$ is the basic law of conduction or dissipation of heat, where k is the thermal conductivity of the medium, P is the area of the cross-section of the medium, **H** is the rate of heat dissipated, $\frac{dI}{dy}$ is the temperature gradient and the negative sign indicates that the heat is transferring in the direction of decreasing temperature. Generally, the temperature distribution and hence the rate of heat convected from the infinite fin surface have been determined via the calculus approach [1-4]. This paper presents for the analysis of uniform infinite fin to obtain the temperature distribution and hence the rate of heat convected into the surroundings by uniform infinite fin.

DEFINITIONS

2.1 Elzaki Transform

If the function $\hbar(y)$, $y \geq 0$ is having an exponential order and is a piecewise continuous function on any interval, then the Elzaki transform of $\hbar(y)$ is given by

$$E\{\hbar(y)\} = \bar{\hbar}(p) = p\int_0^\infty e^{-\frac{y}{p}}\hbar(y)y.$$

The Elzaki Transform [1, 2, 3] of some of the functions are given by

- $E\,\{y^n\} = k!\,p^{n+2}$, $where\ k = 0,1,2,..$
- $E\,\{e^{ay}\} = \frac{p^2}{1-ap}$,
- $E\,\{sikay\} = \frac{ap^3}{1+a^2p^2}$,
- $E\,\{cosay\} = \frac{ap^2}{1+a^2p^2}$,
- $E\,\{sikhay\} = \frac{ap^3}{1-a^2p^2}$,
- $E\,\{coshay\} = \frac{ap^2}{1-a^2p^2}$.

2.2 Inverse Elzaki Transform

The Inverse Elzaki Transform of some of the functions are given by

- $E^{-1}\{p^n\} = \frac{y^{n-2}}{(n-2)!}$, $k = 2, 3, 4 \ldots$
- $E^{-1}\{\frac{p^2}{1-ap}\} = e^{ay}$
- $E^{-1}\{\frac{p^3}{1+a^2p^2}\} = \frac{1}{a}\sin ay$
- $E^{-1}\{\frac{p^2}{1+a^2p^2}\} = \frac{1}{a}\cos ay$
- $E^{-1}\{\frac{p^3}{1-a^2p^2}\} = \frac{1}{a}\sin hay$
- $E^{-1}\{\frac{p^2}{1-a^2p^2}\} = \frac{1}{a}\cos hay$

2.3 Elzaki Transform of Derivatives

The Elzaki Transform [1, 2, 3] of some of the Derivatives of h(y) are given by

- $E\{\hbar'(y)\} = \frac{1}{p}\bar{\hbar}(p) - p\,\hbar(0)$,
- $\{\hbar''(y)\} = \frac{1}{p^2}\bar{\hbar}(p) - \hbar(0) - p\hbar'(0)$,

akd so ok.

FORMULATION

The differential equation which describes the heat dissipated from a uniform infinite fin is given by

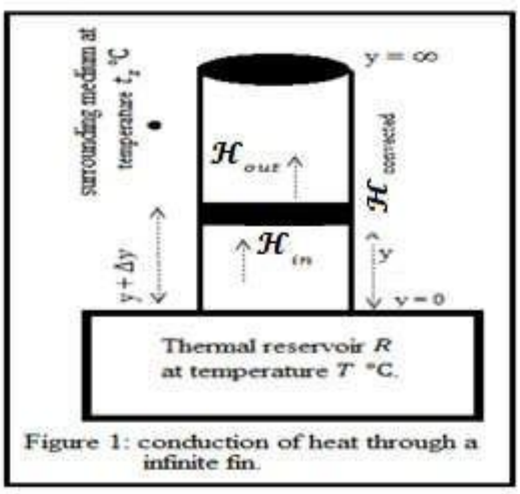

Figure 1: conduction of heat through a infinite fin.

$t''(y) - \frac{\sigma P}{1P}[J(y) - t_s] = 0$ ……… (a), where Let us consider that the one end of the fin is connected to a heat source at y = 0 and the other end at y = ∞ is free for losing heat into the surroundings. The source of heat is maintained at fixed temperature 'T' and t_s is the temperature of the surroundings of the infinite fin and is kept constant.

For convenience, let $(\frac{\sigma P}{1P})^{\frac{1}{2}} = \mu$……. (b)

And $(y) - t_s = \varphi(y)$ … … … (c) known as the excess temperature at the length 'y' of the infinite fin. Then equation (4) can be rewritten as

$\varphi''(y) - \mu^2\,\varphi(y) = 0$ ....(d)

Equations (a) and (d) are the general form of energy equations for one-dimensional heat dissipation from the surface of the infinite fin. In equation (b), μ is a constant provided that σ is constant over the entire surface the infinite fin and k is constant within the range of temperature considered.

The necessary initial conditions are [e, f]

(i) (0) = T. In terms of excess temperature, at y = 0, $J - t_s$ = T- t_s or (0) = φ_0… (e)

(ii) $J(\infty) = t_s$ .In terms of excess temperature, at y = ∞, $\varphi(\infty) = 0$

Taking Elzaki Transform of equation (d), we get

$\frac{1}{q^2}\bar{\varphi}(q) - \varphi(0) - q\varphi'(0) - \mu^2\bar{\varphi}(q) = 0$... (f)

Applying boundary condition: (0) = r_0, equation (f) becomes

$\frac{1}{q^2}\bar{\varphi}(q) - \varphi_0 - q\varphi'(0) - \mu^2\bar{\varphi}(q) = 0$

Or

$\frac{1}{q^2}\bar{\varphi}(q) - \mu^2\bar{\varphi}(q) = q\varphi'(0) + \varphi_0$ ….. (g)

In this equation, $\varphi'(0)$is some constant.

Let us substitute $\varphi'(0) = \omega$,

Equation (g) becomes

$\frac{1}{q^2}\bar{\varphi}(q) - \mu^2\bar{r}(q) = q\omega + \varphi_0$

Or

$\bar{\varphi}(q) = \frac{q^3\omega}{(1-q^2\mu^2)} + \frac{q^2c_0}{(1-q^2\mu^2)}$ ……. (h)

Taking inverse Elzaki Transform of above equation, we get

$\varphi(y) = \frac{\omega}{\mu} sikh\mu y + r_0 \cos h\mu y$

Or

$\varphi(y) = \frac{\omega}{2\mu}[e^{\mu y} - e^{-\mu y}] + \varphi_o[\frac{e^{\mu y} + e^{-\mu y}}{2}]$… (i)

Determination of the constants:

Applying initial condition: $(\infty) = 0$, we can write

$$\frac{\omega}{2\mu}[e^{\mu(\infty)} - e^{-\mu(\infty)}] + \varphi_o[\frac{e^{(\infty)} + e^{-\mu(\infty)}2}{}] = 0$$

Or

$$\frac{\omega}{2\mu}[e^{\beta(\infty)} - 0] + \varphi_o[\frac{e^{(\infty)} + 0}{2}] = 0$$

Or

$$[\frac{\omega}{2\mu} + \frac{\varphi_o}{2}]e^{\mu(\infty)} = 0$$

As $e^{(\infty)} \neq 0$, therefore,

$[\frac{\omega}{2\mu}+\frac{\varphi_0}{2}] = 0$

Or

$\omega = -\mu\varphi_0$.................(j)

Put the value of ω from equation (j) in equation (i), we get

$\varphi(y) = \frac{-\varphi_0}{2\mu}[\, e^{\mu y} - e^{-\mu y}] + r_0\, [\frac{e^{\mu y} + e^{-\mu y}}{2}]$

Or

$\varphi(y) = \frac{-\varphi_0}{2}[\, e^{\mu y} - e^{-\mu y}] + \varphi_0\, [\frac{e^{\mu y} + e^{-\mu y}}{2}]$

Or

$\varphi(y) = \frac{\varphi_0}{2}[\, e^{\mu y} + e^{-\mu y} - e^{\mu y} + e^{-\mu y}]$

Or

$\varphi(y) = \varphi_0 e^{-\mu y}$ ············(k)

Equation (14) provides the distribution of temperature along the length of the infinite fin and confirms that the temperature of the infinite fin decreases along its length with the increase in distance from the heat source maintained at the temperature$T$.

The amount of heat convected from the surface of the infinite fin can be obtained by using the equation [d-f]

$$H_f = -KA\,[D_y J(y)]_{y=0}$$

Or

$H_f = -KA\,[D_y\,(y)]_{y=0}$ …… (l)

Now since $\varphi'(y) = -\mu\varphi_0 e^{-\mu y}$,

Therefore,

$[\varphi'(y)]_{y=0} = -\mu\varphi_0$….. (m)

Using (m) in (l), we get

$V_f = KA\mu\varphi_0$

Or

$V_f = KA\mu\,(T - t_s)$….. (n)

Put the value of μ from equation (6) in equation (17), we get

$V_f = KA\left(\frac{\sigma P}{1P}\right)^{\frac{1}{2}}(T - t_s)$

Or

$V_f = (KA\sigma P)^{\frac{1}{2}}(T - t_s)$ (r)

This equation (r) provides the rate of heat convected from the surface of the infinite fin into its surroundings and confirms that the rate of convection of heat can be increased by increasing the surface area of the fin.

CONCLUSION

In this paper, Elzaki Transform is exemplified for the study of uniform infinite fin for determining the temperature distribution by the side of the infinite fin, and the amount of heat convected from its surface into the environs. We have fulfilled that the temperature of the infinite fin decreases with the increase in its length from the heat source, and the rate of heat convected from the infinite fin surface into the environs can be better by increasing the surface area of the infinite fin.

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