

## ANALYSIS OF UNIFORM INFINITE FIN BY ELZAKI TRANSFORM

Dr Govind Raj Naunyal , Dr Updesh Kumar,

Associate Professor

Department of Mathematics, KGK (PG) College Moradabad

Dr Dinesh Verma

**Abstract:** The amount of heat convected from the fin surfaces has been determined by solving the general differential equation describing heat dissipation from the infinite fin via the calculus approach. Heat transfers by desirable quality of temperature gradient and the modes which transfer heat from one part of the medium to another are conduction, convection, and radiation. This paper is presenting the use of a Elzaki Transform for the analysis of uniform infinite fin by solving the general form of energy equation describing the heat dissipation from the surface of the medium and obtaining the distribution of temperature and hence the rate of heat convected into the surroundings from an infinite uniform fin.

**Keywords:** Heat convected, Uniform Infinite Fin, Elzaki Transform.

### INTRODUCTION

Elzaki Transformation applied in solving boundary value problems in most of the science and engineering disciplines [1, 2, 3, 4, 5, 6, 7]. It also comes out to be very effective tool to analyze differential equations, Simultaneous differential equations, Integral equations etc. [7, 8, 9, 10, 11, 12, 13, 14]. spines are the extended surfaces projected from heat-conducting surfaces to improve the heat dissipation into the surroundings [1-3]. Fourier's law expressed as  $H = -KA \frac{dt}{dy}$ , is the basic law of conduction or dissipation of heat, where  $k$  is the thermal conductivity of the medium,  $A$  is the area of the cross-section of the medium,  $H$  is the rate of heat dissipated,  $\frac{dt}{dy}$  is the temperature gradient and the

negative sign indicates that the heat is transferring in the direction of decreasing temperature. Generally, the temperature distribution and hence the rate of heat convected from the infinite fin surface have been determined via the calculus approach [1-4]. This paper presents for the analysis of uniform infinite fin to obtain the temperature distribution and hence the rate of heat convected into the surroundings by uniform infinite fin.

### DEFINITIONS

#### 2.1 Elzaki Transform

If the function  $h(y)$ ,  $y \geq 0$  is having an exponential order and is a piecewise continuous function on any interval, then the Elzaki transform of  $h(y)$  is given by

$$E\{h(y)\} = \tilde{h}(p) = p \int_0^{\infty} e^{-\frac{y}{p}} h(y) dy.$$

The Elzaki Transform [1, 2, 3] of some of the functions are given by

- $E\{y^n\} = k! p^{n+2}$ , where  $k = 0, 1, 2, \dots$
- $E\{e^{ay}\} = \frac{p^2}{1-ap}$ ,
- $E\{sikyay\} = \frac{ap^3}{1+a^2p^2}$ ,
- $E\{\cos ay\} = \frac{ap^3}{1+a^2p^2}$ ,
- $E\{sikhay\} = \frac{ap^3}{1-a^2p^2}$ ,
- $E\{\cosh ay\} = \frac{ap^3}{1-a^2p^2}$ .

#### 2.2 Inverse Elzaki Transform

The Inverse Elzaki Transform of some of the functions are given by

- $E^{-1}\{p^n\} = \frac{y^{n-2}}{(n-2)!}, k = 2, 3, 4 \dots$
- $E^{-1}\left\{\frac{p^2}{1-ap}\right\} = e^{ay}$
- $E^{-1}\left\{\frac{p^3}{1+a^2p^2}\right\} = \frac{1}{a} \sin ay$
- $E^{-1}\left\{\frac{p^2}{1+a^2p^2}\right\} = \frac{1}{a} \cos ay$
- $E^{-1}\left\{\frac{p^3}{1-a^2p^2}\right\} = \frac{1}{a} \sin hay$
- $E^{-1}\left\{\frac{p^2}{1-a^2p^2}\right\} = \frac{1}{a} \cos hay$

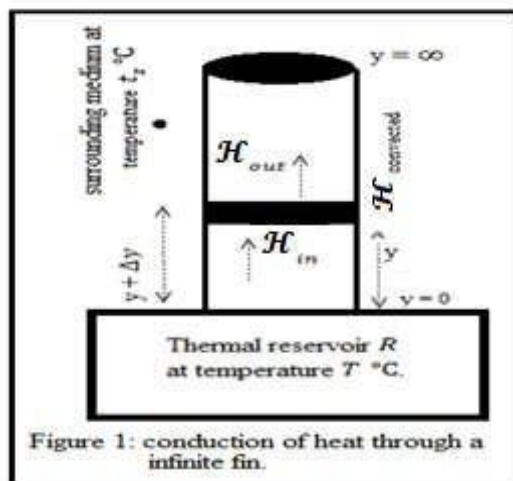
### 2.3 Elzaki Transform of Derivatives

The Elzaki Transform [1, 2, 3] of some of the Derivatives of  $h(y)$  are given by

- $E\{h'(y)\} = \frac{1}{p}h(p) - p h(0),$
- $E\{h''(y)\} = \frac{1}{p^2}h(p) - h(0) - p h'(0),$   
and so on.

### FORMULATION

The differential equation which describes the heat dissipated from a uniform infinite fin is given by



$t''(y) - \frac{\sigma P}{K A} [t(y) - t_s] = 0 \dots\dots\dots (a)$ , where Let us consider that the one end of the fin is connected to a heat source at  $y = 0$  and the other end at  $y = \infty$  is free for losing heat into the surroundings. The source of heat is maintained at fixed temperature  $T$  and  $t$  is

the temperature of the surroundings of the infinite fin and is kept constant.

For convenience, let  $\left(\frac{\sigma P}{K A}\right)^{\frac{1}{2}} = \mu \dots\dots\dots (b)$

And  $t(y) - t_s = \varphi(y) \dots\dots\dots (c)$  known as the excess temperature at the length 'y' of the infinite fin. Then equation (4) can be rewritten as

$$\varphi''(y) - \mu^2 \varphi(y) = 0 \dots\dots\dots (d)$$

Equations (a) and (d) are the general form of energy equations for one-dimensional heat dissipation from the surface of the infinite fin. In equation (b),  $\mu$  is a constant provided that  $\sigma$  is constant over the entire surface the infinite fin and  $k$  is constant within the range of temperature considered.

The necessary initial conditions are [e, f]

(i)  $t(0) = T$ . In terms of excess temperature, at  $y = 0, t - t_s = T - t_s$  or  $\varphi(0) = \varphi_0 \dots\dots (e)$

(ii)  $t(\infty) = t_s$ . In terms of excess temperature, at  $y = \infty, \varphi(\infty) = 0$

Taking Elzaki Transform of equation (d), we get

$$\frac{1}{q^2} \bar{\varphi}(q) - \varphi(0) - q \varphi'(0) - \mu^2 \bar{\varphi}(q) = 0 \dots\dots (f)$$

Applying boundary condition:  $\tau(0) = \tau_0$ , equation (f) becomes

$$\frac{1}{q^2} \bar{\varphi}(q) - \varphi_0 - q \varphi'(0) - \mu^2 \bar{\varphi}(q) = 0$$

Or

$$\frac{1}{q^2} \bar{\varphi}(q) - \mu^2 \bar{\varphi}(q) = q \varphi'(0) + \varphi_0 \dots\dots (g)$$

In this equation,  $\varphi'(0)$  is some constant.

Let us substitute  $\varphi'(0) = \omega$ ,

Equation (g) becomes

$$\frac{1}{q^2} \bar{\varphi}(q) - \mu^2 \bar{\tau}(q) = q \omega + \varphi_0$$

Or

$$\bar{\varphi}(q) = \frac{q^3 \omega}{(1 - q^2 \mu^2)} + \frac{q^2 \tau_0}{(1 - q^2 \mu^2)} \dots\dots\dots (h)$$

Taking inverse Elzaki Transform of above equation,

we get

$$\varphi(y) = \frac{\omega}{\mu} \sinh \mu y + \tau_0 \cosh \mu y$$

Or

$$\varphi(y) = \frac{\omega}{2\mu} [e^{\mu y} - e^{-\mu y}] + \varphi_0 \left[ \frac{e^{\mu y} + e^{-\mu y}}{2} \right] \dots\dots (i)$$

### Determination of the constants:

Applying initial condition:  $\tau(\infty) = 0$ , we can write

$$\frac{\omega}{2\mu} [e^{\mu(\infty)} - e^{-\mu(\infty)}] + \varphi_0 \left[ \frac{e^{\mu(\infty)} + e^{-\mu(\infty)}}{2} \right] = 0$$

Or

$$\frac{\omega}{2\mu} [e^{\beta(\infty)} - 0] + \varphi_0 \left[ \frac{e^{\mu(\infty)} + 0}{2} \right] = 0$$

Or

$$\left[ \frac{\omega}{2\mu} + \frac{\varphi_0}{2} \right] e^{\mu(\infty)} = 0$$

As  $e^{\mu(\infty)} \neq 0$ , therefore,

$$\left[\frac{\omega}{2\mu} + \frac{\varphi_o}{2}\right] = 0$$

Or

$$\omega = -\mu\varphi_o \dots\dots\dots (j)$$

Put the value of  $\omega$  from equation (j) in equation (i), we get

$$\varphi(y) = \frac{-\varphi_o}{2\mu} [e^{\mu y} - e^{-\mu y}] + \tau_o \left[\frac{e^{\mu y} + e^{-\mu y}}{2}\right]$$

Or

$$\varphi(y) = \frac{-\varphi_o}{2} [e^{\mu y} - e^{-\mu y}] + \varphi_o \left[\frac{e^{\mu y} + e^{-\mu y}}{2}\right]$$

Or

$$\varphi(y) = \frac{\varphi_o}{2} [e^{\mu y} + e^{-\mu y} - e^{\mu y} + e^{-\mu y}]$$

Or

$$\varphi(y) = \varphi_o e^{-\mu y} \dots\dots\dots (k)$$

Equation (14) provides the distribution of temperature along the length of the infinite fin and confirms that the temperature of the infinite fin decreases along its length with the increase in distance from the heat source maintained at the temperature  $T$ .

The amount of heat convected from the surface of the infinite fin can be obtained by using the equation [d-f]

$$H_f = -KA [D_y \varphi(y)]_{y=0}$$

$$\text{Or } H_f = -KA [D_y \varphi(y)]_{y=0} \dots\dots (l)$$

$$\text{Now since } \varphi'(y) = -\mu\varphi_o e^{-\mu y},$$

Therefore,

$$[\varphi'(y)]_{y=0} = -\mu\varphi_o \dots\dots (m)$$

Using (m) in (l), we get

$$H_f = KA\mu\varphi_o$$

Or

$$H_f = KA\mu (T - t_s) \dots\dots (n)$$

Put the value of  $\mu$  from equation (6) in equation (17), we get

$$H_f = KA \left(\frac{\sigma P}{K\mathcal{A}}\right)^{\frac{1}{2}} (T - t_s)$$

Or

$$H_f = (KA\sigma P)^{\frac{1}{2}} (T - t_s) \quad (r)$$

This equation (r) provides the rate of heat convected from the surface of the infinite fin into its surroundings and confirms that the rate of convection of heat can be increased by increasing the surface area of the fin.

## CONCLUSION

In this paper, Elzaki Transform is exemplified for the study of uniform infinite fin for determining the temperature distribution by the side of the infinite fin, and the amount of heat convected from its surface into the environs. We have fulfilled that the temperature of the infinite fin decreases with the increase in its length from the heat source, and the rate of heat convected from the infinite fin surface

into the environs can be better by increasing the surface area of the infinite fin.

## REFERENCES

- [1] Dinesh Verma, Elzaki –Laplace Transform of some significant Functions, Academia Arena, Volume-12, Issue-4, April 2020..
- [3] Dinesh Verma, Aftab Alam, Analysis of Simultaneous Differential Equations By Elzaki Transform Approach, Science, Technology And Development Volume Ix Issue I January 2020.
- [4] Sunil Shrivastava, Introduction of Laplace Transform and Elzaki Transform with Application (Electrical Circuits), International Research Journal of Engineering and Technology (IRJET), volume 05 Issue 02, Feb-2018.
- [5] Tarig M. Elzaki, Salih M. Elzaki and Elsayed Elnour, On the new integral transform Elzaki transform fundamental properties investigations and applications, global journal of mathematical sciences: Theory and Practical, volume 4, number 1(2012).
- [6] Dinesh Verma and Rahul Gupta, Delta Potential Response of Electric Network Circuit, Iconic Research and Engineering Journal (IRE) Volume-3, Issue-8, February 2020.
- [7] D.S. Kumar, Heat and mass transfer, (Seventh revised edition), Publisher: S K Kataria and Sons, 2013.
- [8] P.K. Nag, Heat and mass transfer. 3<sup>rd</sup> Edition. Publisher: Tata McGraw-Hill Education Pvt. Ltd., 2011.
- [9] Rohit Gupta, Amit Pal Singh, Dinesh Verma, Flow of Heat through A Plane Wall, And Through A Finite Fin Insulated At the Tip, International Journal of Scientific & Technology Research, Vol. 8, Issue 10, Oct. 2019, pp. 125-128.
- [10] Rohit Gupta, Neeraj Pandita, Rahul Gupta, Heat conducted through a parabolic fin via Means of Elzaki transform, Journal of Engineering Sciences, Vol. 11, Issue 1, Jan. 2020, pp. 533-535.
- [11] Rohit Gupta, On novel integral transform: Rohit Transform and its application to boundary value problems, ASIO Journal of Chemistry, Physics, Mathematics and Applied Sciences, 2020, 4(1): 08-13.
- [12] Rohit Gupta, Rahul Gupta, Matrix method approach for the temperature distribution and heat flow along a conducting bar connected between two heat sources, Journal of Emerging Technologies and Innovative Research, Vol. 5 Issue 9, Sep. 2018, pp. 210-214.
- [13] Rohit Gupta, Rahul Gupta, Heat Dissipation From The Finite Fin Surface Losing Heat At The Tip, International Journal of Research and Analytical Reviews, Vol. 5, Issue 3, Sep. 2018, pp. 138-143.
- [14] Rohit Gupta, Rahul Gupta, Dinesh Verma, Laplace Transform Approach for the Heat Dissipation from an Infinite Fin Surface, Global Journal Of Engineering Science And Researches, 6(2) February 2019, pp. 96-101.